Hypergeometric Probability

What is **Hypergeometric Probability** ?

It is a method to compute probabilities when the selection is done from two different groups with following criteria:

- Each group contains different items.
- We select items without replacement.
- Order of arrangements does not matter.

If these requirements are satisfied, then organize the following chart

	Group 1	Group 2	Total
Total Objects	n_1	n_2	$n_1 + n_2$
Selected Objects	r_1	r_2	$r_1 + r_2$

Then

$$\mathbf{P}(\mathbf{r_1} \& \mathbf{r_2}) = \frac{{}_{\mathbf{n_1}}\mathbf{C}_{\mathbf{r_1}} \cdot {}_{\mathbf{n_2}}\mathbf{C}_{\mathbf{r_2}}}{{}_{(\mathbf{n_1}+\mathbf{n_2})}\mathbf{C}_{(\mathbf{r_1}+\mathbf{r_2})}}$$

Example:

In a recent city election, all registered voters were supposed to vote for 5 city counsel members. There were 14 candidates, 5 females and 9 males. What is the probability that 3 females and 2 males were selected?

Solution:

We first make our chart,

	Females	Males	Total
Total Objects	5	9	14
Selected Objects	3	2	5

Now we apply the hypergeometric probability formula

$$P(3F\&2M) = \frac{{}_5C_3 \cdot {}_9C_2}{{}_{14}C_5} = \frac{180}{1001}$$

Example:

In a city sponsored lottery game, you must select 4 numbers in any order from a list of numbers ranging from 1 to 20. The fundraisers draw 4 numbers randomly as winning numbers. What is the probability that you have all 4 winning numbers?

Solution:

We first make our chart,

	Winning Numbers	Losing Numbers	Total
Total Objects	4	16	20
Selected Objects	4	0	4

Now we apply the hypergeometric probability formula

$$P(4W\&0L) = \frac{{}_4C_4 \cdot {}_{16}C_0}{{}_{20}C_4} = \frac{1}{4845}$$

Example:

A box contains 6 red balls and 14 black balls. Suppose you are blindfolded and randomly select 3 different balls. What is the probability that you have selected 3 red balls?

Solution:

We first make our chart,

	Red Balls	Black Balls	Total
Total Objects	6	14	20
Selected Objects	3	0	3

Now we apply the hypergeometric probability formula

$$P(3R\&0B) = \frac{{}_{6}C_{3} \cdot {}_{14}C_{0}}{{}_{20}C_{3}} = \frac{1}{57}$$

Example:

A box contains 6 red balls and 14 black balls. Suppose you are blindfolded and randomly select 3 different balls. What is the probability that you have selected 3 black balls?

Solution:

We first make our chart,

	Red Balls	Black Balls	Total
Total Objects	6	14	20
Selected Objects	0	3	3

Now we apply the hypergeometric probability formula

$$P(0R\&3B) = \frac{{}_6C_0 \cdot {}_{14}C_3}{{}_{20}C_3} = \frac{91}{285}$$

Example:

In California Super Lotto, we must choose 5 numbers from 1 to 47 for the winning numbers, and one number from 1 to 27 for the winning mega number, see the image below.

What is the probability of having all 5 winning numbers and have the winning mega number as well?

Solution:

Since this procedure takes place without replacement and order does not matter, 5 winning numbers are chosen from a list of 47 numbers, and the winning mega number is chosen from a list of 27 numbers, We first make our chart,

	47 Regular Numbers		27 Mega Numbers	
	W Number	L Number	W Mega	L Mega
Total	5	42	1	26
Selected	5	0	1	0

Now, let's compute the total number of ways that this can be done

$$\begin{aligned} {}_{47}C_5 \cdot {}_{27}C_1 &= 1533939 \cdot 27 \\ &= 41,416,353 \text{ different ways} \end{aligned}$$

Solution Continued:

Now for our desired event of selecting 5 winning numbers and the winning mega number, we get

$$\begin{aligned} {}_5C_5 \cdot {}_{42}C_0 \cdot {}_1C_1 \cdot {}_{26}C_0 &= 1 \cdot 1 \cdot 1 \cdot 1 \\ &= 1 \text{ way} \end{aligned}$$

And now we are ready to find the probability for our desired event,

$$\text{Probability(5W, 0L, 1 WM)} = \frac{1}{41416353}$$

Example:

Consider California Super Lotto, what is the probability of having exactly 3 winning numbers and a losing mega number?

Solution:

With exactly 3 winning numbers, there is also 2 losing numbers to be included as well, along with a losing mega number, We first make our chart,

	47 Regular Numbers		27 Mega Numbers	
	W Number	L Number	W Mega	L Mega
Total	5	42	1	26
Selected	3	2	0	1

Now for the number of ways for our desired event to occur, we get

$$\begin{aligned} {}_5C_3 \cdot {}_{42}C_2 \cdot {}_1C_0 \cdot {}_{26}C_1 &= 10 \cdot 861 \cdot 1 \cdot 26 \\ &= 223860 \text{ ways} \end{aligned}$$

And now we are ready to find the probability for our desired event,

$$\text{Probability(3W, 2L, 1 LM)} = \frac{223860}{41416353} = \frac{74620}{13805451}$$

Example:

A coin jar contains 3 quarters, 5 dimes, and 12 nickels. Assume a blindfolded person grabs 3 coins from this jar, what is the probability that this person has collected 40 cents?

Solution:

The only way to get a total of 40 cents while drawing 3 coins is to have one of each type of coin, now We can make our chart,

	Quarter	Dime	Nickel	Total
Total	3	5	12	20
Selected	1	1	1	3

Now, let's compute the total number of ways that this can be done

$$ {}_{20}C_3 = 1140 \text{ different ways} $$

Solution Continued:

Now for the number of ways for our desired event to occur, we get

$$\begin{aligned} {}_3C_1 \cdot {}_5C_1 \cdot {}_{12}C_1 &= 3 \cdot 5 \cdot 12 \\ &= 180 \text{ different ways} \end{aligned}$$

And now we are ready to find the probability for our desired event,

$$\begin{aligned} \text{Probability(40 cents)} &= \text{Probability(1Q, 1D, 1N)} \\ &= \frac{180}{1140} \\ &= \frac{3}{19} \end{aligned}$$